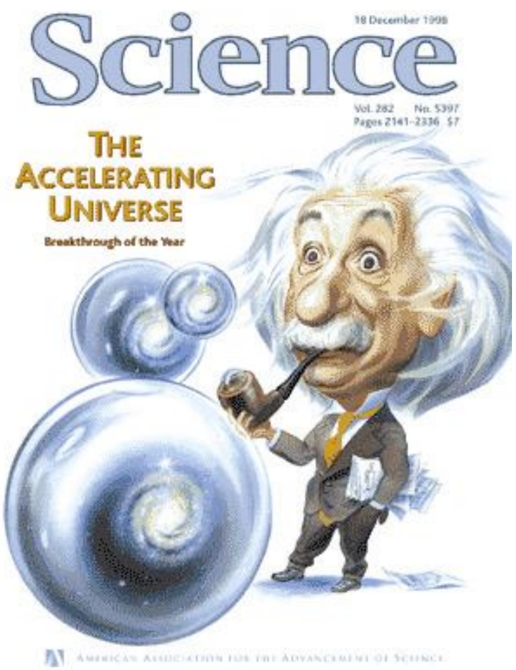
A Very Brief, Very Simple Introduction to
Dark Energy



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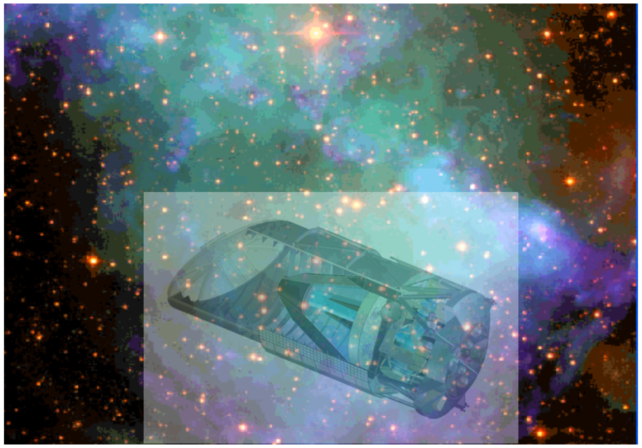

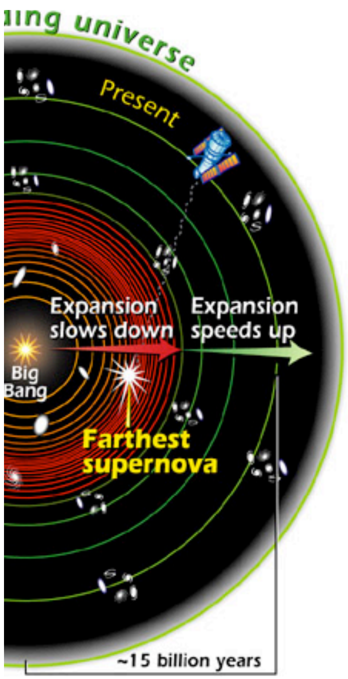

Acceleration and Dark Energy

Einstein says gravitating mass depends on energy-momentum tensor:
both energy density ρ and pressure p, as $\rho+3p$

Negative pressure can give negative "mass"

Newton's 2nd law: Acceleration = Force / mass

$$\ddot{R} = -GM/R^2 = -(4\pi/3)G\,\rho\,R$$

Einstein/Friedmann equation:

$$\ddot{a} = -(4\pi/3)G\,(\rho+3p)\,a$$

Negative pressure can accelerate the expansion

Negative pressure

Relation between ρ and p (***equation of state***) is crucial:

$$\boxed{w = p / \rho}$$

Acceleration possible for $p < -(1/3)\rho$ or $w < -1/3$

What does negative pressure mean?

Consider 1st law of thermodynamics:

$$dU = -p\, dV$$

But for a spring $dU = +k\, x dx$

or a rubber band $dU = +T\, dl$

Vacuum Energy

Quantum physics predicts that the very structure of the vacuum should act like springs.

Space has a "springiness", or tension, or vacuum energy with negative pressure.

Review --
***Einstein*: expansion acceleration depends on $\rho+3p$**
***Thermodynamics*: pressure p can be negative**
***Quantum Physics*: vacuum energy has negative p**

Supernova markers can map the expansion history, measure acceleration, detect vacuum energy.

What Dark Energy Isn't

Is dark energy related to dark matter? Probably not.

DM: clumps around galaxies and clusters
DE: smoothly distributed

DM: pressureless, dominated by rest mass
DE: strongly negative pressure, of order energy density

DM: important to evolution of universe since z_{eq}: t=100,000 y
DE: important to evolution of universe since t≈7 billion y

DM: interacts through gravity and (presumably) weak force
DE: interacts only through gravity

DM: makes up 20% of total energy density today, 70% at z_{dec}
DE: makes up 75% now, negligible amount at z_{dec}

What Dark Energy Isn't

Is dark energy related to inflation? Probably not.

Inflation: at energy scale around 10^{24} eV
DE: at energy scale around 10^{-3} eV

[Energy density goes as (scale)^4, making the ratio 10^{108}.]

Inflation: began at perhaps $t \approx 10^{-35}$ s
DE: acceleration began at $t \approx 7$ billion y

Is dark energy related to Casimir energy? Probably not.

Casimir: depends on physical boundaries
DE: universe has no physical boundaries

Casimir: can have positive pressure, depending on boundary geometry
DE: negative pressure, universe has no physical boundaries

Casimir: detected effect arises from vector field (EM)
DE: not a vector field (no charge), maybe scalar field

Casimir: dies off rapidly with distance ($\sim d^{-4}$)
DE: repulsion (tension) increases with distance

Dark Energy

Dark energy is likely to be a fundamentally new physical phenomenon.

For more dark energy resources, see http://supernova.lbl.gov/~evlinder/scires.html and the *Resource Letter on Dark Energy* http://arxiv.org/abs/0705.4102

Cosmology Formulas

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Densities:

$$\rho_w(z) = \rho_w(0)\, e^{3\int_0^{\ln 1+z} d\ln(1+z)\,[1+w(z)]} \equiv \rho_w(0)\, f_w(z). \tag{1}$$

Hubble parameter:

$$\begin{aligned} H^2(z) &= \frac{8\pi G}{3} \sum \rho_w(z) - ka^{-2} && (2) \\ &= H_0^2 \left[\sum \Omega_w(0)\, f_w(z) + (1-\Omega_T)\, a^{-2}\right] && (3) \end{aligned}$$

where the total dimensionless density $\Omega_T = \sum \Omega_w(0)$ and the dimensionless density $\Omega_w(0) = 8\pi G\rho_w(0)/(3H_0^2)$. Note $k = (\Omega_T - 1)a_0^2 H_0^2$ is the spatial curvature constant, $a = a_0/(1+z)$, and one cannot simultaneously choose a and k dimensionless, e.g. $a_0 = 1$ *and* $k = \pm 1$.

Distances:

$$r_c(z) = \int dt/a = H_0^{-1}(1-\Omega_T)^{-1/2} \sinh[(1-\Omega_T)^{1/2} \int_0^z dz/(H/H_0)] \tag{4}$$

is the **comoving distance**. This expression is analytically valid for $\Omega_T = 0$, > 1, or < 1.

$$r_p(z) = \int dt = \int_0^z dz/[(1+z)\, H(z)] \tag{5}$$

is the **proper distance**.

Three distances based on observational methods are the angular diameter distance r_a, proper motion distance r_m, and luminosity distance r_l. In a general cosmology these are not given in terms of r_c or r_p, but within any metric theory of gravity do share the interrelationship

$$r_l = (1+z)\, r_m = (1+z)^2\, r_a. \tag{6}$$

This is closely related to the second law of thermodynamics (see Linder 1988) and so is called the thermodynamic relation. In a metric theory of gravity it is broken only if the phase space density of photons is not conserved (violation of Liouville's theorem). Note that recently the relation between r_l and r_a has been called the reciprocity relation; that in fact refers to a related relation between $r_a(z,z')$ and $r_a(z',z)$. Within general relativity the expression for any one of them comes from a second order differential equation. Within Friedmann-Robertson-Walker cosmologies, the situation simplifies considerably in that $r_m = r_c$.

Volume:

$$dV = r_a^2 \, dr_p \, d\omega \tag{7}$$

is the proper volume element, where $d\omega$ is the solid angle on the sky. The proper volume is just the integral of this. The comoving volume element $dV_c = (1+z)^3 dV$.

Note that a measurement of an angular scale θ within the plane of the sky (transverse to line of sight) probes $\theta = L/r_a$, where L is the physical scale. A measurement parallel to the line of sight probes $\Delta r = \int_x^{x+L} dr_p$. Only when the physical scale is truly infinitesimal does this reduce to $\Delta r \sim 1/[(1+z)\,H(z)]$.

Growth:

For a mass density perturbation $\delta \equiv \delta\rho/\rho$, the growth within general relativity (and where all matter and only matter is perturbed) is governed by a second order differential equation

$$\ddot{\delta} + 2H\dot{\delta} - (3/2)\Omega_m(a)H^2\delta \;=\; 0 \tag{8}$$

$$\delta'' + (2-q)a^{-1}\delta' - (3/2)\Omega_m(a)\,a^{-2}\delta \;=\; 0 \tag{9}$$

where dot denotes a time derivative and prime a derivative with respect to scale factor a. The deceleration parameter $q = -a\ddot{a}/\dot{a}^2 = -1 - d\ln H/d\ln a$.

The (growing) solution for a pure matter universe is $\delta \sim a$, and observations of large scale structure indicate that our universe must have been substantially matter dominated for $z \approx 2-1000$, so it is convenient to define a normalized growth variable $g \equiv \delta/a$. The equation for this matter density growth variable, when all matter and only matter is perturbed, is

$$g'' + \left[4 + \frac{1}{2}(\ln H^2)'\right] g' + \left[3 + \frac{1}{2}(\ln H^2)' - \frac{3}{2}\Omega_m(a)\right] g = 0 \tag{10}$$

Here a prime is derivative with respect to $\ln a$. The matter density $\Omega_m(a) = \Omega_m a^{-3}/[H/H_0]^2$.